

Hanle Effect near Boundaries

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The Hanle effect describes suppression of spin polarization due to precession in a magnetic field. This is a standard spintronics tool and it gives access to the spin lifetime of samples in which spins are generated homogeneously. We examine the Hanle effect when spins are generated at a boundary of a diffusive sample by the extrinsic spin Hall effect. We show that the Hanle curve is spatially dependent and that the “apparent” spin lifetime, given by its inverse half-width, is shorter near the boundary even if the spin relaxation rate is homogenous.

The goal of spintronics is to generate and manipulate spin populations on time scales limited by the spin lifetime. One can access the spin population optically, since selection rules allow optical pumping and detection of spins in materials [1]; interesting alternatives are magnetic materials or materials with spin-orbit interaction, providing access to spins with standard microelectronic devices [2, 3]. To characterize a given sample, it is essential to determine its spin lifetime τ_s , which depends on the microscopic properties of the sample. One can determine τ_s of a homogeneous sample using the Hanle effect [1] as follows, even if time-resolved measurements are not available. If there is no spin precession, a spin polarization simply decays with time τ_s . However, if a magnetic field B perpendicular to the spin polarization axis is applied, there is a competing relaxation mechanism: spins will precess in that magnetic field with Larmor frequency $\omega_L \propto B$. If the magnetic field is sufficiently large, such that the spin can precess many times within its lifetime, this will randomize the spin and suppress the spin polarization. This competing spin relaxation mechanism becomes effective for $\omega_L \gtrsim 1/\tau_s$ —thus τ_s can be extracted by measuring the inverse width of the so-called Hanle curve $s_z(\omega_L)$.

In recent experiments by Kato *et al.* [4], a spatially dependent spin polarization s_z was induced via the extrinsic spin Hall effect [5, 6, 7, 8] and measured via Kerr microscopy. The width of the Hanle curves $s_z(\omega_L, \mathbf{r})$ was described with a spatially-dependent spin lifetime $\tilde{\tau}_s(\mathbf{r})$. Rather strikingly, it was found that $\tilde{\tau}_s$ is several times smaller near the sample edge than 10 μm away from the edge. In this article we calculate the Hanle curves and show that such a suppression of $\tilde{\tau}_s$ near the edge can result from spin diffusion, even if the spin relaxation rate τ_s^{-1} is spatially *homogeneous*.

The physical picture for this spatial dependence of $\tilde{\tau}_s$ is as follows [see Fig. 1(a),(b)]. Spins are generated at the boundary and then diffuse into the bulk of the sample. In a magnetic field, the spins observed at a small distance x were (on average) generated a short time ago and did not yet precess much in the magnetic field. Therefore, they have a larger s_z than one would expect for the homogeneous case with a bulk generation mechanism (e.g., optical pumping). This means that the linewidth as function of B is larger and the spin lifetime seems smaller. Conversely, the spins observed far from the boundary, re-

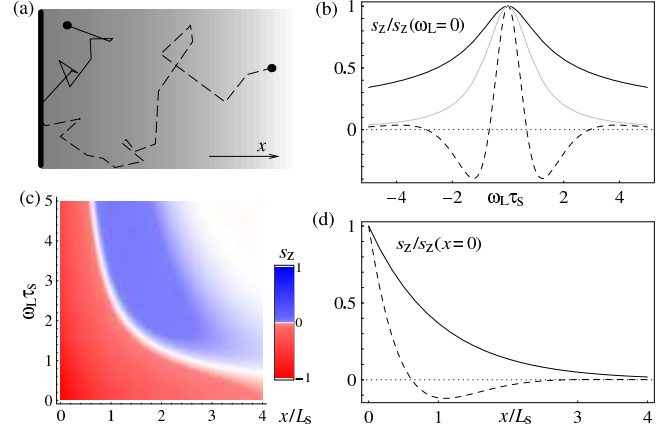


Figure 1: Spin density $s_z(x, \omega_L)$ near a boundary of a diffusive system, given by Eq. (6) for $\tau_s = \tau_{xy} = \tau_z$ and $q_s = 0$. (a) When spins are generated at the boundary and then diffuse into the sample, spins closer to the boundary had on average less time to precess in the magnetic field $\mathbf{B} = B\hat{\mathbf{x}}$ than those further away. (b) Therefore, the Hanle curve $s_z(\omega_L \propto B)$ close to the boundary ($x = 0$, solid line) becomes broader, while away from the boundary ($x = 4L_s$, dashed line) it becomes narrower than in the case of homogeneous spin generation (gray line). (c) Sufficiently far from the boundary the Hanle curve develops “sidelobes” [9, 10] where the spin polarization changes sign; here s_z is plotted in units of $j_x^z \sqrt{\tau_s/D}$. (d) Spins generated at the boundary diffuse into the sample in the absence of a magnetic field and eventually become suppressed due to spin relaxation ($\omega_L = 0$, solid line), while in a magnetic field ($\omega_L = 5\tau_s^{-1}$, dashed line) spin precession further suppresses spin polarization.

quired a rather long time to get there and were able to precess longer in the magnetic field. Therefore, the value of s_z is more strongly suppressed by B , the linewidth becomes narrower, and the spin lifetime appears longer.

A similar situation is found when the dominating spin transport mechanism is the drift induced by charge currents [11, 12, 13, 14]. During the drift from the injection to the detection point over a distance r , spins precess during time $t = r/v_{\text{dr}}$, where v_{dr} is the drift velocity. Because the precession angle $\omega_L t$ is the same for each spin (neglecting diffusion), multiple oscillations of s_z were observed as function of ω_L [11] or r [12].

To quantitatively describe the suppression of the apparent spin lifetime $\tilde{\tau}_s$ in a diffusive system, we now

analyze the Hanle curves for such systems. For this, we consider a magnetic field $\mathbf{B} = B\hat{\mathbf{x}}$, which induces spin precession of electrons with Larmor frequency $\omega_L = g^*\mu_B B/\hbar$, with effective g -factor g^* and Bohr magneton μ_B , corresponding to a Zeeman coupling $H_Z = \frac{1}{2}g^*\mu_B\mathbf{B}\cdot\boldsymbol{\sigma}$. We assume a sufficiently small magnetic field that orbital effects are not important and that τ_s^{-1} is independent of B . The equation of motion for the spin density \mathbf{s} , including spin precession, diffusion, and relaxation is

$$\dot{\mathbf{s}} = (g^*\mu_B/\hbar)\mathbf{B}\times\mathbf{s} + D\Delta\mathbf{s} - \overleftarrow{\tau}_s^{-1}\mathbf{s}, \quad (1)$$

with a spatially independent spin diffusion constant D and a diagonal spin relaxation tensor $\overleftarrow{\tau}_s^{-1}$ with components $\{\tau_{xy}^{-1}, \tau_{xy}^{-1}, \tau_z^{-1}\}$; note that the polarization s_x decouples and so its spin relaxation rate is actually not important here. Also, we define the geometrical mean of the spin relaxation times as $\tau_s = \sqrt{\tau_{xy}\tau_z}$ and the spin diffusion length is $L_s = \sqrt{D\tau_s}$. We set $\tau_{xy} = \alpha\tau_s$ and $\tau_z = \alpha^{-1}\tau_s$ with some dimensionless constant α , e.g., $\alpha = \sqrt{2}$ for Dyakonov-Perel spin relaxation and Rashba coupling [15, 16, 17, 18].

Next we assume that spin polarization is generated at a boundary plane. This is the case for the extrinsic spin Hall effect [4, 5, 6, 7, 8], where an electrical current induces spin currents which in turn produce spin polarization near sample edges due to extrinsic spin-orbit interaction. We take a semi-infinite two- or three-dimensional system with $x \geq 0$, and an electric field E_y applied along the y -direction. The transverse spin current is $j_x^z = \sigma_{\text{SH}}E_y$, with spin Hall conductivity σ_{SH} ; microscopically the spin current relaxes on the short transport lifetime $\tau \ll 1/\omega_L$, thus σ_{SH} does not depend on the weak magnetic field. If spin is conserved at the boundary, there is no spin current perpendicular to the boundary and the spin Hall current is compensated by spin diffusion, i.e., $j_x^z = D\frac{\partial}{\partial x}s_z$ at $x = 0$. More generally, we consider the boundary condition

$$\frac{\partial}{\partial x}s_z = \frac{j_x^z}{D} + q_s s_z, \quad \frac{\partial}{\partial x}s_y = q_s s_y, \quad (2)$$

which allows for spin relaxation at the edge, characterized by q_s , and where we have taken $j_x^y = 0$.

For other systems, where spins are generated at a boundary and then precess in a field, Eqs. (1), (2) also apply and these systems show the same Hanle curves. D'yakonov and Perel' [19] considered the situation where electron spins were optically generated using circularly polarized light in a surface layer thinner than L_s . Assuming that recombination only takes place in this surface layer, it is taken into account via q_s . Further, the degree of circular polarization of the recombination radiation is proportional to s_z at $x = 0$, so only the Hanle curve at the surface is experimentally accessible. Such measurements were reported by Vekua *et al.* [20]. Furthermore, Johnson and Silsbee [10], analyzed a system where spins are injected from a ferromagnet into a paramagnet at $x = 0$. A second ferromagnet at a distance

x is used as a detector, whose voltage is proportional to the spin polarization $s_z(x)$. Fabrication of devices with different detector spacings then provides electrical access to the spatially-dependent Hanle curve.

We now analyze the spin polarization in the stationary case $\dot{\mathbf{s}} = 0$ by assuming that the spin relaxation rate $\overleftarrow{\tau}_s^{-1}$ is *spatially independent*. With the ansatz $\mathbf{s} = \mathbf{s}_0 e^{qx}$ we find the solutions of Eq. (1) satisfying $\text{Re } q < 0$,

$$q_{0,1} = -\sqrt{\frac{\tau_{xy} + \tau_z \pm T}{2D\tau_{xy}\tau_z}}, \quad (3)$$

$$T = \sqrt{(\tau_{xy} - \tau_z)^2 - 4\tau_{xy}^2\tau_z^2\omega_L^2}. \quad (4)$$

From the boundary conditions (2), we obtain the position-dependent Hanle curves

$$s_y(x, B) = j_x^z \sum_{i=0,1} e^{q_i x} \frac{(-1)^i \tau_{xy}\tau_z\omega_L}{DT (q_i - q_s)}, \quad (5)$$

$$s_z(x, B) = j_x^z \sum_{i=0,1} e^{q_i x} \frac{T + (-1)^i (\tau_{xy} - \tau_z)}{2DT (q_i - q_s)}. \quad (6)$$

For $\tau_{xy} = \tau_z = \tau_s$, Eq. (6) simplifies considerably; using $q_{0,1}L_s = -\sqrt{1 \pm i\omega_L\tau_s} = -(\kappa_R \pm i\kappa_I)$ with $\kappa_R = (1 + \sqrt{1 + \omega_L^2\tau_s^2})^{1/2}/\sqrt{2}$ and $\kappa_I = \omega_L\tau_s/2\kappa_R$, we find

$$s_z(x) = -\frac{j_x^z \sqrt{\tau_s} [(\kappa_R + \beta) \cos \kappa_I d + \kappa_I \sin \kappa_I d]}{\sqrt{D} [(\kappa_R + \beta)^2 + \kappa_I^2]} e^{-\kappa_R d}, \quad (7)$$

where we have defined $\beta = q_s L_s$. In the special case of $x = 0$ and $\tau_{xy} = \tau_z$, Eq. (7) agrees with the expression found when studying Hanle effect on surfaces [19, 20]; while for $q_s = 0$ and $\tau_{xy} = \tau_z$ it agrees with the result from Ref. 10.

Further, in the absence of the magnetic field, Eq. (6) simplifies to $s_z = -j_x^z \sqrt{\tau_s/D} e^{-\sqrt{\alpha}x/L_s}/(\sqrt{\alpha} + \beta)$. Finally, for $q_s = 0$, the integrated spin density corresponds to the Hanle curve of a homogeneous system,

$$\int dx s_z(x) = -\frac{j_x^z \tau_z}{1 + \tau_s^2 \omega_L^2}. \quad (8)$$

In the experiments of Ref. 4, the apparent spin lifetime $\tilde{\tau}_s(x)$ is extracted by assuming a Lorentzian Hanle curve [Eq. (8)] at each position, then τ_s can be found as the half-width at half-maximum of $s_z(\omega_L)$. Correspondingly, we now take Eq. (6) and solve $\frac{1}{2}s_z(x, \omega_L = 0) = s_z(x, \omega_L^{\text{HWHM}})$ for the apparent spin lifetime $\tilde{\tau}_s(x) = 1/\omega_L^{\text{HWHM}}$. Since $\tilde{\tau}_s$ does not depend on the prefactor j_x^z in s_z , it is a function of $\alpha, \beta, \tau_s, x, D$. From dimensional analysis we see that

$$\tilde{\tau}_s = \tau_s g_{\alpha,\beta} \left(\frac{x}{L_s} \right), \quad (9)$$

with some dimensionless function $g_{\alpha,\beta}$ that depends on the distance $d = x/L_s$ from the boundary in units of the spin diffusion length.

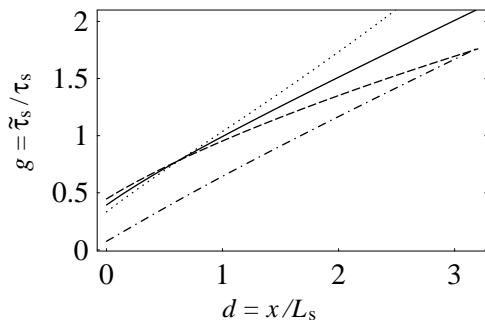


Figure 2: Apparent spin relaxation time $\tilde{\tau}_s$ as function of distance x from boundary, plotted for $\beta = 0$ and $\alpha = 1$ (solid line), $\alpha = \sqrt{2}$ (dashed), $\alpha = 1/\sqrt{2}$ (dotted) and for $\beta = 2$ and $\alpha = 1$ (dashed-dotted). While details depend on the microscopic parameters (see text), generally the apparent spin lifetime is reduced when considering the Hanle curves close to the boundary, even if the spin relaxation rate is position-independent.

Using Eq. (6), we evaluate $g_{\alpha,\beta}(d)$ numerically and plot it in Fig. 2. For example, $g_{1,0} \approx 0.43 + 0.52d$ within 5% and for $d < 10$. Most importantly, the “apparent” spin relaxation time $\tilde{\tau}_s$ shows a strong spatial dependence, even if the underlying spin relaxation rate is homogeneous. In particular, this means that $\tilde{\tau}_s$ is roughly *four times smaller* near the boundary than several (three to four) spin diffusion lengths away. This is in agreement with the experiments [4, 13, 21] where a similar factor was observed [22].

Furthermore, note that at finite distances x , the Hanle curve can develop “sidelobes,” where s_z changes sign, see Fig. 1. This is a well-known effect and such sidelobes were detected electrically in Johnson-Silsbee geometries for a fixed injector-detector spacing x [23, 24]. Additionally, in the regime $\tau_{xy} \gg \tau_z$ and for a fixed x , the polarization at finite fields can have a larger magnitude (but opposite sign) than the main peak at zero fields, which can be understood as follows. In the absence of spin precession (main peak), the spins will relax rapidly with rate τ_z^{-1} . However, the precessing spins corresponding to the sidelobes relax with a lower average rate and thus contribute with a larger signal, effectively filtering spins that have precessed by an angle of π .

An important question is what happens at the boundary of a homogeneous sample, namely if there are spin relaxations processes due to the boundary. Such processes, on length scales shorter than L_s , are included here via q_s . By measuring the spatially dependent Hanle curves and by fitting with Eq. (6) (or by comparing with Fig. 2), one can extract q_s and therefore gain access to the relaxation at the boundary, even if it occurs on a much shorter length scale than spatial resolution of $s_z(x)$. Finally, for an inhomogeneous sample, a local probe of the spin lifetime is desirable. While it is now clear that spin diffusion can make such a measurement difficult in the steady state, one could instead use a time-resolved

(pump-probe) measurement to determine $\tau_s(x)$.

Instead of extracting the parameters of Eq. (1) and (2) by fitting to Eq. (6), one can find some parameters more directly as follows. First note that for $B = 0$, one can extract the decay length $L_s^z = \sqrt{D\tau_z}$ from $s_z(x)$. Next, the width of the Hanle curve contains information about spin relaxation, and we access it via the curvature at the origin, $c(x) = (\partial^2 s_z / \partial B^2) / s_z|_{B=0}$. (The normalization of c eliminates effects of a spatially dependent detection sensitivity on s_z .) Since the Hanle curve becomes narrower when moving away from the boundary, the curvature increases and from Eq. (6), we find

$$\frac{1}{c} \frac{\partial c}{\partial x} \Big|_{x=0} = \frac{1}{L_s^z} + q_s, \quad (10)$$

which does not explicitly depend on α or g^* . Because L_s^z can be determined independently, Eq. (10) provides a convenient way to access the spin relaxation q_s at the boundary.

In addition to the extrinsic spin-orbit interaction, leading to the spin Hall effect considered above, there is also intrinsic spin-orbit interaction that couples to the spin as an effective field $\mathbf{b}(\mathbf{k})$, depending on the wave vector \mathbf{k} . In Eq. (1), we do not take this field into account explicitly; however, it does contribute to the spin relaxation rate τ_s^{-1} . Also, this field can lead to additional spin polarization induced by the electric field—for example, for a two-dimensional system with Rashba spin-orbit interaction, this polarization is along the x axis [25, 26, 27, 28, 29, 30, 31]; however, it is not relevant in our discussion of s_z , since s_x does not couple to s_z or s_y in Eq. (1). In a naive model, one can understand this polarization as arising from the field $\mathbf{b}_{\text{dr}} = \langle \mathbf{b}(\mathbf{k}) \rangle$ averaged over all carriers, which drift in the electric field with a finite $\langle \mathbf{k} \rangle$. For Rashba spin-orbit interaction, \mathbf{b}_{dr} is in-plane and perpendicular to \mathbf{E} , i.e., in our case $\mathbf{b}_{\text{dr}} \parallel \mathbf{B}$.

In addition to the s_x polarization, \mathbf{b}_{dr} contributes as a spin precession term in the Bloch equation. Because it is parallel to \mathbf{B} , its contribution can be absorbed into ω_L and it leads to a shifted Hanle curve $s_z(B)$ with the maximum moved away from $B = 0$. Experimentally, the expected shift of the Hanle curve $s_z(B)$ was reported for strained three-dimensional n -GaAs systems (where a spin-orbit coupling with the same form as the Rashba term is present [32]), while for unstrained samples one sees that $\mathbf{b}_{\text{dr}} = 0$ due to the cubic symmetry and there is no shift [4]. Note that this naive model can break down for more general transport mechanisms [31], which can lead to spin generation along $\mathbf{b}_{\text{dr}} \times \mathbf{B}$, but this expression vanishes in our configuration.

Furthermore, for Rashba spin-orbit interaction there are additional precession terms around the \hat{y} axis that arise when spins diffuse away from the edge [17, 33, 34, 35]. This would induce oscillations in $s_z(x)$ in addition to the one shown in Fig. 1(d) and the combined effect can lead to larger oscillation amplitudes. Since the precession length is on the order of L_s in both cases, strictly speaking our model [Eq. (1)] does not apply to a system

with Rashba spin-orbit interaction—however, no such k -linear intrinsic spin-orbit terms are present for a three-dimensional system with cubic symmetry, which applies to the experiments of Ref. 4 on unstrained samples. Finally, for two-dimensional systems, it was argued that the Rashba spin-orbit interaction can change the magnitude of extrinsic spin currents [36, 37] and would thus change the magnitude of the Hanle curves. For these systems, also the importance of the intrinsic spin-orbit interaction on the boundary conditions was studied [38, 39]; measuring the spatial dependence of the Hanle curves and using a property analogous to Eq. (10) can be used to test these predictions.

In conclusion, we have found that in systems where spins are generated at the boundary, the magnetic field dependence of the spin polarization (Hanle curve) be-

comes spatially dependent even if the spin relaxation rate τ_s^{-1} is spatially homogenous. This leads to a reduction of the “apparent” spin lifetimes $\tilde{\tau}_s$ near the edges of a sample exhibiting the spin Hall effect, as was recently observed experimentally [4]. We have provided an intuitive picture for this effect: spins detected closer than L_s to the edge were on average generated within a time less than τ_s and relatively large magnetic fields would be required to suppress them, corresponding to a small $\tilde{\tau}_s$. Our description provides a method for extracting the homogeneous spin relaxation rate and it also allows to measure spin relaxation effects at the sample boundary.

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